

Hydrodynamic instability of the shocked water/gas interface ‡

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Abstract

Hydrodynamic instability and resulting interfacial turbulent mixing are studied in a liquid/gas shock-tube configuration. HYDRA, an AWE three dimensional Eulerian Massively Parallel Processing (MPP) code, is used to numerically investigate such phenomena. In addition, the cavitation formation and its effect on the turbulent mixing, by adding a small amount of gas pre-mixed with water, are presented and discussed. ‡

Introduction

Richtmyer-Meshkov instability (RMI) occurs when different density or compressibility fluids with interfacial perturbations are subject to an impulse acceleration, *e.g.* a shock wave [1, 2, 3, 4]. RMI, together with other forms of hydrodynamic instabilities, is a fundamental fluid phenomenon, consisting of a non-stationary and nonlinear physical process. It occurs in a wealth of man-made applications and natural events, such as, Inertial Confinement Fusion (ICF), Supernova explosions and many combustion systems. It is therefore of great importance to better comprehend the dynamics of such instabilities. In this study, we are concerned with interfacial instabilities and turbulent mixing characteristics of a heavy liquid (*e.g.* Water) and light gas (Air) within a shock-tube setting.

To gain a more thorough understanding of the dynamic physics and mixing characteristics, we base our models on a simplified geometry of a shock-tube to investigate the interactions between shocks and material interface, fluctuations and formation of complex small-scale structures. The presence of gas cavities further adds the complex of fluid interactions and the mixing dynamics. Numerically simulating such flow phenomena is very challenging.

The interfacial turbulent mixing problems are, in general, highly dependent on small scale perturbations. In the case of nearly pure liquid, *e.g.* the mixture of liquid and a small amount of gas (less than 0.01 in volume fraction), cavitation may occur. To simulate such flows, it is desirable to use very fine meshes for features of interest which may only occupy a small region of the physical domain, yet advance in space as the flow evolves. We apply a Eulerian code HYDRA, utilizing the Monotonic Integrated Large Eddy Simulation (MILES) strategy, which has demonstrated a great success in similar shock-interface interaction and mixing processes of RMI/RTI problems at AWE. Built upon this MILES framework, we further extend the code ability by adding a one-dimensional extension option and the moving mesh functionality for an efficient solution of interest.

Numerical methods

Hydra solver

Hydra is a three-dimensional Eulerian Massively Parallel Processing (MPP) code, developed for the solution of multi-

materials hydrodynamical problems, with density and internal energy of each fluid separately solved. It uses a Lagrange and re-map (or advection) methodology, such that every time cycle is divided into a Lagrangian phase, in which the mesh moves with the fluid velocity, and a re-mapping phase where the fluid is moved back to the original fixed Eulerian frame. In Lagrangian phase the changes in velocity and internal energy due to pressure terms are calculated. The sum of the kinetic and internal energies is conserved. While the re-mapping phase calculates the transport of mass, internal energy and momentum across cell boundaries with the monotonic advection method, which minimizes numerical diffusion yet preventing spurious numerical oscillation. Partial volume fluxes are calculated with an interface reconstruction algorithm. In addition, a semi-Lagrangian moving mesh facility and an alternative volume fraction advection algorithm are implemented for an efficient and accurate solution of highly mixed flows.

A form of the stiffened gas equation of state (EoS) is selected for water and gas:

$$P_k(\rho_k, e_k) = (\gamma_k - 1)\rho_k e_k - \gamma_k P_{\infty, k}$$

where, the parameters: $\gamma_{water}=4.4$, $P_{\infty, water} = 6 \times 10^8$ Pa, $\gamma_{gas} = 1.8$ and $P_{\infty, gas} = 0$ Pa, as referred in [5].

Computational domain and boundary conditions

The configuration of interest is filled with water in the left and gas in the right (pure or nearly pure) separated by a pre-described profile or randomly-perturbed surface, as shown in Figure 1. A quasi-3D meshing was designated in the region of $0 \leq x \leq 2$ m. To reduce the computational cost, we use one-dimensional extension to 6 m on the right. A moving meshing facility is used to track the movement of dynamic waves and interesting features, minimizing numerical dissipation. The left and the *y*-sidewalls of the shock tube are computationally, a reflective boundary. The *z*-sidewalls in the simulations is a periodic boundary condition.

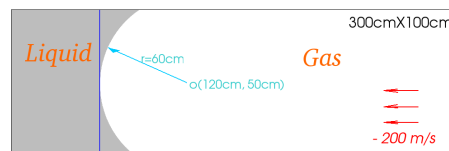


Figure 1: The computation domain

Initial conditions and perturbations

The flow field is initialized in two regions: a high density of 1000 kg/m^3 liquid on the left and a light gas of 100 kg/m^3 on the right. They are at thermal and pressure equilibrium, with an initial pressure of one atmospheric pressure (10^5 Pa) and velocity of 200 m/s (to the left direction). It is separated by a curved

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interface, which is a portion of circle with a radius of 0.6 m centered at the point (1.2 m, 0.5 m).

In the validation below, an initial perturbation representation, similar to the one in [3], is used to incorporate symmetry-breaking modes and random micro-scale irregularities on an straight line materials interface.

Flow characteristics

Upon the flow impacted on the left wall at a location of $x=0$ m, a right-facing shock at $M_s \approx 1.20$ is generated with a pressure of $\sim 3.8 \times 10^8$ Pa and propagates to the water/gas discontinuity of the domain. Refracted on the interface at a time of ~ 0.3 ms, a transmitted shock and reflected rarefaction waves are then generated. A conventional RMI appears first, then expansion waves are produced near the wall as the jet elongates with a dynamic appearance of cavitation.

At a late time, these shocks and waves are reflected from evolving interface and the walls, with further interactions increasing fluctuations and formation of complex small-scale structures. The time evolution of mixing layer properties and turbulent characteristics is also affected by the propagation and interactions of these shock/rarefaction waves and the material interfaces, as schematically illustrated in Figure 2. A range of

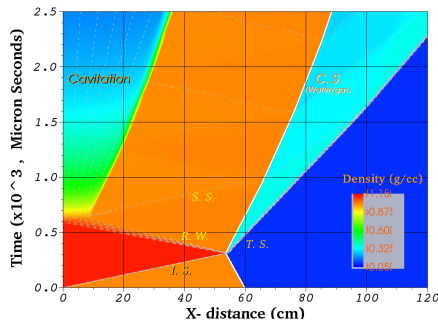


Figure 2: The X-T wave diagram.

meshes were generated to study grid-independence and ensure a converged solution. Unless specified, simulated results by a mesh of $2400 \times 400 \times 4$ are presented and analyzed.

Validation

For the purpose of the validation, first we numerically investigate the same configuration and flow conditions as described before, but with a straight membrane instead of circle one, as shown in blue line of Figure 1. It was intended to make a comparison with a theoretical analysis and experimental observations if available.

Figure 3 shows the evolution of the mixture density. An incident shock is initiated and propagates towards the right direct, upon an instant impact of the left moving fluid on the wall. At times of 0.25 and 0.40 ms, we present the amplitude of initial perturbations and growth just before and shortly after the incident shock arrives the water/gas interface, respectively. The structure of bubbles and spikes and their developments are evidently nonlinear as time evolves to 1.0 ms and 1.6 ms. It becomes turbulent at late times of 2.5 ms and 4.0 ms, respectively. As the left-facing expansion wave reflected from the wall, the expanded water is accelerated towards the right, and creates a very lower density and pressure region from 1.5 ms onwards.

To investigate two-phase effects, we conduct a similar simulation with nearly pure fluids, with a very lower volume fraction gas (water) evenly distributed in original pure water (gas), re-

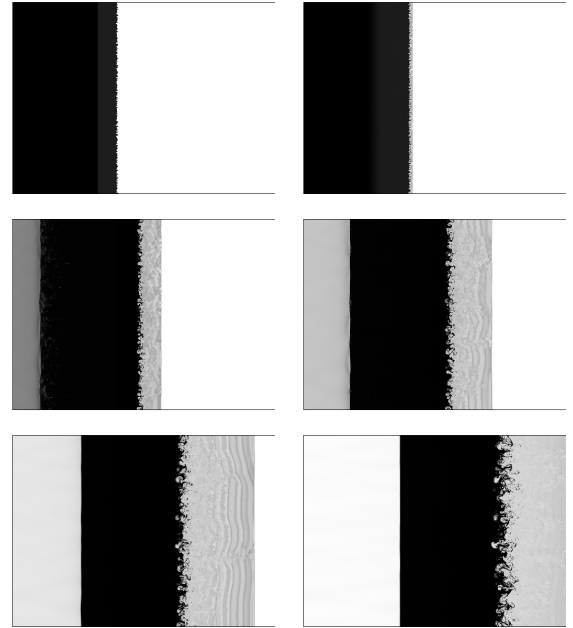


Figure 3: The evolution of the density at times of 0.25, 0.40, 1.0, 1.6, 2.5 and 4.0 ms, respectively.

spectively. In Figure 4, we produce the mixture density plots at times of 1.6 ms and 4.0 ms when a volume fraction of 0.001 is pre-mixed in each side of the shock-tube. Consequently, Atwood number ($At = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$) is slightly decreases to 0.8165455 from 0.8181818 by 0.2%.



Figure 4: The evolution of the mixture density at times of 1.6 (Left) and 4.0 (Right) ms, respectively.

Note that identical perturbations are imposed on the materials interface in both simulations. The perturbed density pattern of gas behind the transmitted shock is clearly different at a time of 1.6 ms. It is well-known that a thinner mixing zone is expected by the reduction of Atwood number from the RMI theory. On the contrary, longer spikes are observed at 4.0 ms.

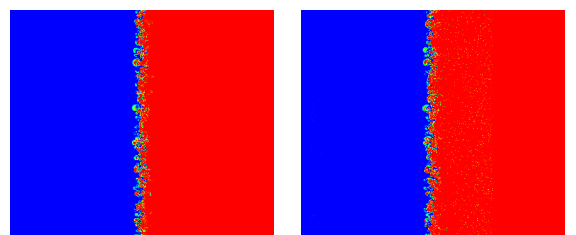


Figure 5: Comparison of the gas volume fraction between pure (Left) and nearly pure (Right) fluids at a time of 2.0 ms.

Figure 5 presents a direct comparison of simulated volume frac-

tion at a time of 2.0 ms. It is not obvious shown here that a small amount of gas (water) shown for nearly pure case. Nevertheless, the shape and size of the spikes and bubbles are visibly different due to fluid species modified in each side of the interface.

It is further confirmed by a comparison of the mixing layer width between pure and nearly pure cases, as made in Figure 6. The mixing zone on the spike side is argued by the presence of water droplets in the gas volume.

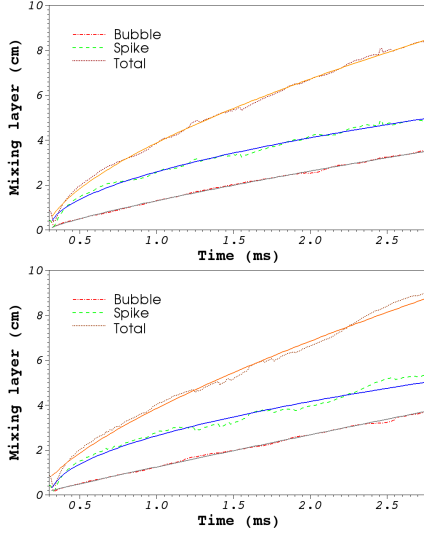


Figure 6: Comparison of the mixing layer width between pure and nearly pure fluids.

RMI nonlinear growth theory for multimode perturbations states that the temporal evolution of the width of the turbulent mixing zone (TMZ) follows a power law $h \propto t^\theta$, where the θ values ranging from 0.25 to 1 have been documented [1]. Indeed the amplitude for the bubbles and spikes can be fit (the solid lines in Figure 6) by the following model equation:

$$h(t) = h(0) \left(1 + \frac{dh(0)}{dt} \frac{t}{h(0)\theta} \right)^\theta$$

We find that $\theta_b \simeq 0.5$ produces a reasonable fit to the quasi-3D simulation data for the bubble growth for the both cases. While a high value of θ_s ($\simeq 0.94$) is required for a better fit of spike TMZ growth in nearly pure liquid (gas) case, compared to $\theta_s \simeq 0.80$ of the pure case.

Results and discussion

Now we turn our attention to the curved interface as shown in Figure 1, which provides us rich features of the shock refraction on the materials interface and resulting turbulent mixing.

Figure 7 shows the evolution of the mixture density at times of 0, 0.6, 1.0, 2.0, 3.1, 4.8, 6.4 and 8.6 ms, respectively. One can clearly see that the evolution of a liquid jet, an incident shock and its refraction: a transmitted shock and reflected rarefaction waves, and the size and shape of bubbles and spikes forming and growing on the interface.

Note that there is no surface roughness perturbation applied. As shown at a time of 0 ms, a curved material surface projected in a Cartesian orthogonal mesh numerically creates a perturbed but axisymmetric (along the central line) material interface, which is intrinsically related to the mesh resolution.

Upon the instant impact on the left-hand wall, an incident shock

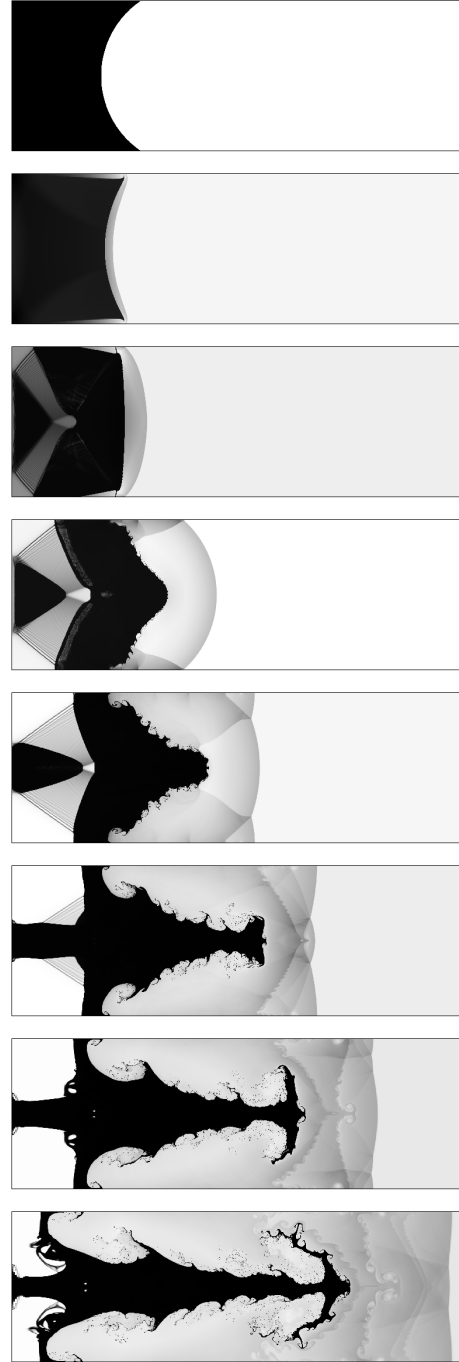


Figure 7: The evolution of the mixture density of pure liquid at times of 0, 0.6, 1.0, 2.0, 3.1, 4.8, 6.4 and 8.6 ms, respectively.

is initiated and propagates into the right. A conventional RMI occurs as the shock interacts with the material interface between the heavy fluid to light gas. The curvature of interface inverts as time proceeds to 0.6 ms. It results in expansion zones near the solid wall where the gas flow in-homogeneities grows. Consequently a core spike structure is formed as a liquid jet. Then expansion waves are produced as the jet elongates, producing dynamic gas cavities near the wall and in the jet core, clearly shown in figures at times of 1.0 ms and 2.0 ms. During these processes, the hydrodynamic instabilities grow progressively at the material interface, and develop in to a fully turbulent mixing flow leading to a symmetry breaking, as shown from a time of 3.1 ms in the figure.

There are several salient features in this dynamic mixing process. Firstly a secondary jet starts emerging at the front of the core jet and grows as the time evolves. Secondly, we can see that a Mach stem exists near the wall as a result of interaction between a curved transmitted shock and the solid wall. A reflected shock branch of the Mach stem interacts further with the core liquid jet, forming a narrow jet head part as shown at 4.8 ms. The core jet and secondary jet continue to grow and finally a typical mushroom shape of a nonlinear spike growth appears as shown at late times of 6.4 ms and 8.6 ms. Thirdly a series of wave patterns, owing to various interactions among shocks, expansion waves, the material interface, the solid wall, is illustrated as a synthetic shadowgraph image in front of the black core liquid jet. We can observe the spiky structure rolling clearly at 6.4 ms. Interestingly, the late arriving waves mimic interface instabilities due to the nature of perturbed shock/expansion waves.

In Figure 8, we present the evolution of the mixture density for nearly pure liquid case, *i.e.*, 0.0001 volume fraction gas (water) pre-mixed with water (gas). Compared with their counterparts of Figure 7, longer spiky fingers are clearly seen on the material interface at a time of 2.0 ms. The Mach stem appears earlier to accommodate a pressure balance. The jet dynamics is modified compared to the conventional RMI with pure liquid case at times of 4.8 and 6.4 ms. The rolling structures shown in the shocked gas region disappear. Instead, a slender jet forms. The shock position, the size and shape of bubbles and spikes growth on the water/gas interface are distinguished, too. The head part of the core jet spreads wider and the tip of mushroom thinner and longer. The formation and dynamics of the gas cavitation are greatly modified near the solid boundary and in the core jet due to a very low pressure in these regions. In general, the

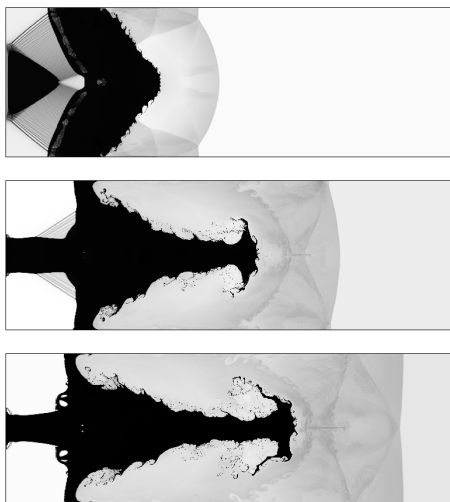


Figure 8: The evolution of the mixture density of nearly pure liquid at times of 2.0, 4.8 and 6.4 ms, respectively.

results shown here are comparable to those predicted with a compressible multiphase Godunov method by Saure *et al.* [5]

Figure 9 compares the volume fraction of water for an initial gas (water) volume of 0, 0.0001 and 0.01 mixture in each side of the shock tube, respectively. The tip position and shape of the core liquid jet, the size and shape of bubbly and spiky structures near the material interface, and appearance of cavitation differ significantly. As the volume of initial gas (water) increases, the length of the core jet and spikes on the water/gas interface are compressed. Meanwhile, the cavitation volume increases as the water expands. Similarly, water coalesces as the shock

propagates in the nearly pure gas portion of the shock-tube.

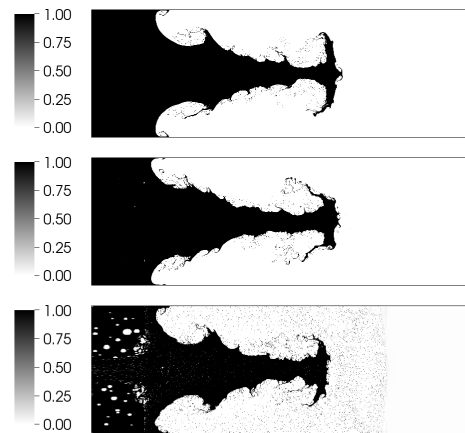


Figure 9: Comparison of the volume fraction contours at a time of 6.4 ms between 0, 0.0001 and 0.01 mixture gas liquid.

It is worthwhile mentioning that in all three cases, the interface perturbations are exactly identical due to the use of the same grid discretization. The change in Atwood number is negligible (less than 2%). Theoretically, no noticeable changes in the flow pattern and mixing characteristics are observed. It is demonstrated that the presence of a small amount of gas mixture with water, or likewise water mixture with gas, influences the RMI, turbulent mixing and cavitation significantly.

Conclusions

We have numerically conducted a comprehensive investigation on Richtmyer-Meshkov instability (RMI) characteristics and resulting interfacial mixing properties, obtained from HYDRA simulations of a liquid/gas shock-tube system. The evolution of density and volume fraction contours and the turbulent mixing length width are presented to qualitatively describe a typical turbulent mixing process for both pure and nearly pure water/gas cases. It is demonstrated that a small amount of gas or water mixture significantly influences the RMI characteristics, turbulent mixing and gas cavitation.

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